Localizing differences in smooths with simultaneous confidence bounds on the true discovery proportion

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Abstract

We demonstrate a method for localizing where two smooths differ using a true discovery proportion (TDP) based interpretation. The procedure yields a statement on the proportion of some region where true differences exist between two smooths, which results from its use of hypothesis tests on groups of basis coefficients underlying the smooths. The methodology avoids otherwise ad hoc means of doing so such as performing hypothesis tests on entire smooths of subsetted data. TDP estimates are 1-α confidence bounded simultaneously, assuring that the estimate for a region is a lower bound on the proportion of actual difference, or true discoveries, in that region with high confidence, regardless of the number or location of regions estimated. Our procedure is based on closed-testing [Hommel, 1986] and recent results of Goeman and Solari [2011] and Goeman et al [2019]. We develop expressions for the covariance of quadratic forms because of the multiple regression framework in which we use the closed-testing procedure, which are shown to be non-negative in many settings. Our method is well-powered because of a given result on the off-diagonal decay structure of the covariance matrix of penalized B-splines of degree two or less. We demonstrate achievement of estimated TDP and nominal type 1 error rates in simulation and analyze a data set of walking gait of cerebral palsy patients.
Keywords: splines; smoothing; multiple testing; closed-testing; simultaneous confidence